



A Stabilized Modified Lax Scheme with Numerical Filter for Accurate One-Dimensional Pollutant Transport Modeling

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Abstract

Pollutant transport in water is a global issue commonly modeled using advection-diffusion equations that describe transport driven by concentration gradients and flow velocity. Because analytical solutions are limited to simple cases, numerical methods are essential for simulating pollutant transport. However, many existing numerical models require complex formulations to achieve accurate, stable solutions. This study proposes a one-dimensional numerical model based on a modified Lax scheme combined with the Hansen filter, providing stable, accurate solutions with a simpler formulation. The proposed model is evaluated using three test cases: pure advection, advection-diffusion, and a Gaussian pulse. The performance of the proposed numerical model is compared with the exact solution using L_2 , L_∞ , and absolute error analysis. For the pure advection case, the proposed model achieves $L_2=0.048$ and $L_\infty=0.011$, which are lower than those reported by some previous numerical models. In the advection-diffusion case, the model also has better accuracy than some previous numerical models with $L_2=0.0948$ and $L_\infty=0.0422$. For the Gaussian pulse case, the absolute error remains very small at 8.76×10^{-5} at the concentration peak. show that the proposed model can suppress numerical oscillations while maintaining high accuracy and efficiency, making it effective for one-dimensional simulations of pollutant transport.

Keywords: Advection, Diffusion, Finite Difference, Numerical, Pollutant

INTRODUCTION

The spread of pollutants from domestic, industrial, and agricultural activities [1], [2] across various media, such as water, has become an increasingly important environmental issue. Various problems can arise from the spread of these pollutants, including decreased water quality, ecosystem damage, and disruptions to human health [3]. Therefore, understanding the patterns and characteristics of pollutant pollution in various media, such as water, is crucial for environmental management. Mathematical or numerical modeling plays a crucial role in describing and understanding the patterns and characteristics of pollutant spread in various

media. Two main mechanisms generally drive pollutant transport [4], [5]. The first mechanism is pollutant transport driven by fluid mass movement [6], and the second is transport driven by concentration gradients [7]. These two processes are generally described mathematically by the advection-diffusion equation, which previous researchers have widely used [7], [8], [9].

The advection-diffusion equation has exact solutions only for simple cases, limiting its application to various practical problems. As a result, numerical methods are required to solve more realistic and complex problems of pollutant transport. To overcome this limitation, various numerical models have been

developed to solve the advection-diffusion equation. Various numerical models have been developed [10], [11], [12], [13]. Sanjaya & Mungkasi [14] used the finite difference method to solve the advection-diffusion equation, but artificial oscillations persisted in the simulation results when the spatial step was too large. This instability reduces the accuracy and reliability of the numerical solution. To overcome these oscillations, a stabilization technique such as a numerical filter is required. A finite element method-based model, namely the Finite Element Method using linear shape functions (FEMLSF) and the Finite Element Method using quadratic shape functions (FEMQSF), was developed by [15]. However, the accuracy of the resulting solutions is still relatively low. These results indicate that improving stability alone is insufficient without maintaining high solution accuracy. Another numerical model developed by [16] uses the Sixth-Order Combined Compact Finite Difference Scheme (CD6), which provides more accurate results than other models. However, the CD6 model has a more complex formulation than conventional finite-difference models. The complexity of the formulation increases computational difficulty and reduces implementation efficiency. Therefore, a relatively simple model that produces stable and accurate solutions is needed for one-dimensional pollutant transport modeling. This gap constitutes the main problem addressed in this study.

Based on these needs, this study proposes a numerical model using a modified Lax scheme with an additional Hansen filter. Modifications to the Lax scheme are made by adding additional parameters to control model stability [17]. In addition, the modified Lax scheme is also combined with a Hansen numerical filter to improve model accuracy, as shown by [18] and [19]. The novelty of this study lies in combining the modified Lax scheme and the Hansen filter to develop a numerical model that is simultaneously simple, stable, and accurate. This study aims to evaluate the performance of the proposed model in producing stable and accurate solutions to pollutant transport problems. Model performance is evaluated through testing on several one-dimensional cases, including pure advection, advection-diffusion, and a Gaussian pulse. Simulation

results of the proposed model are compared with exact solutions and other numerical models using error indicators, including norm errors (L_2 and L_∞) and absolute errors, to assess its accuracy and stability.

METHODS

Model Development Methodology

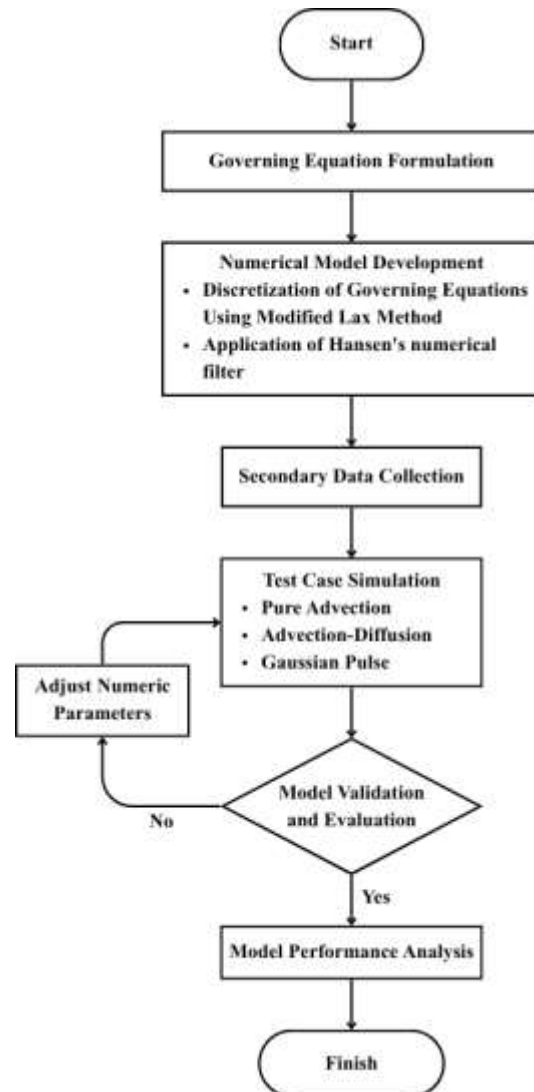


Figure 1. Model Development Flow Chart

In this study, a numerical model was developed through several steps (Figure 1). The first step is to formulate the governing equations that represent the pollutant transport phenomenon. The second step is to develop the numerical model by discretizing the governing equations using the modified Lax method and applying the Hansen filter to the resulting discretized equations. The third step is to collect data from

previous studies for use in the test case simulations. Next, the developed model is tested through several simulation cases, namely pure advection, advection-diffusion, and a Gaussian pulse. The simulation results are then validated and evaluated by comparing them with analytical solutions and several other numerical models using error indicators such as L_2 , L_∞ , and absolute error. If the model's accuracy and stability do not yield good results, adjustments are made to the numerical parameters (time step, spatial step, and the Lax additional parameter), and the simulation is repeated. The final step is a model performance analysis to assess the proposed model's performance to produce accurate, stable solutions for one-dimensional pollutant transport problems.

Research Data

In this study, data were obtained from various previous studies related to pollutant transport modelling. The purpose of using this secondary data is to ensure that the proposed numerical model can be directly compared with and validated against other numerical models from previous studies and exact solutions. These data include channel length, diffusion coefficient, initial flow velocity, and the initial position of the pollutant concentration peak. The channel length is used as the boundary of the computational domain in the simulations. Furthermore, the diffusion coefficient describes the process of pollutant distribution driven by differences in concentration. The initial flow velocity and the initial position of the pollutant concentration peak are used to describe the initial conditions for the pollutant transport simulation.

In addition to physical parameters, this study also uses exact solutions and boundary conditions available in previous studies. The exact solutions are used to evaluate the performance of the proposed numerical model quantitatively. This evaluation compares the pollutant concentration profiles generated by the proposed numerical model with the analytical solutions. Furthermore, the boundary conditions used in this study are consistent with those in previous studies. Based on data from previous studies, several simulation scenarios were developed to test the

performance of the proposed numerical model for one-dimensional pollutant transport.

Governing Equation

The one-dimensional advection-diffusion equation is widely used to describe pollutant transport in a one-dimensional domain. This equation describes two main mechanisms that occur in the pollutant transport process. First, advection, which describes the movement of pollutants due to flow velocity. Second, diffusion, which describes the process of pollutant spread due to differences in pollutant concentration. The following expression gives the advection-diffusion equation used as the governing equation in this study.

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} = D \frac{\partial^2 C}{\partial x^2} \quad (1)$$

Where $C(x, t)$ is the pollutant concentration at position x and time t , u is the flow velocity, D is the diffusion or dispersion coefficient, x is the spatial coordinate, and t is time.

In the one-dimensional advection-diffusion equation, there is a term $\frac{\partial C}{\partial t}$ which describes the change in pollutant concentration over time, $u \frac{\partial C}{\partial x}$ which describes the pollutant transport process due to flow or advection, and $D \frac{\partial^2 C}{\partial x^2}$ which describes the pollutant transport process due to the diffusion process along the flow direction.

Numerical Model

In this study, the governing equations are solved numerically using a finite difference scheme, namely the modified Lax Scheme. The spatial domain will be divided into several grids with a distance of Δx , and the time domain will be divided into intervals Δt . The pollutant concentration value at spatial location i and specific time t is expressed as C_i^n . The Modified Lax scheme is a development of the Lax scheme by adding a parameter (α) to control the stability and accuracy of the resulting solution. The parameter α lies between 0 and 1. When $\alpha = 1$, the scheme will return to the conventional Lax scheme, while

the scheme becomes the Forward Time Central Space (FTCS) scheme when $\alpha = 0$. The following is the result of solving the governing equations using the Modified Lax scheme.

$$C_i^{n+1} = \alpha \left(\frac{C_{i+1}^n + C_{i-1}^n}{2} \right) + (1 - \alpha)C_i^n - \frac{u\Delta t}{2\Delta x}(C_{i+1}^n - C_{i-1}^n) + D \frac{\Delta t}{\Delta x^2}(C_{i+1}^n - 2C_i^n + C_{i-1}^n) \quad (2)$$

During implementation, the stability of the proposed numerical model will depend on the parameter α . In this study, a trial-and-error approach will be used to determine the parameter α to obtain an accurate and stable solution. In addition, determining the grid size (Δx) and the appropriate time step (Δt) is important for obtaining stable, accurate results. The influence of the grid size and time step is generally expressed by a dimensionless parameter, namely the Courant number, as shown in equation (3). Generally, a stable solution will be obtained when the Courant number (CFL) is less than one.

$$CFL = \frac{u\Delta t}{\Delta x} \quad (3)$$

Numerical Filter

To address oscillations that often occur in numerical models, this study uses the Hansen numerical filter as an additional step after the main calculation using the Modified Lax Scheme at each time step. The Hansen filter works by filtering the solution value at each computation point using a combination of the value at that point and the values from surrounding points. This process has been shown to improve the numerical model's stability, as demonstrated by [19]. Furthermore, the Hansen numerical filter has a relatively simple formulation, making it easy to implement in a numerical model.

The filtering process is performed after the concentration value C_i^{n+1} is obtained from the solution process using the Modified Lax Scheme. This value is then filtered using the

following equation to obtain the corrected concentration value:

$$\tilde{C}_i^{n+1} = \beta C_i^{n+1} + (1 - \beta) \left(0.5 \times \frac{C_{i+1}^n + C_{i-1}^n}{2} \right) \quad (4)$$

Where \tilde{C}_i^{n+1} is the filtered concentration value and C_i^{n+1} is the concentration value before filtering. β is the Hansen filter parameter, set to 0.99. This value has been shown to provide stable results in previous studies [19], [20], [21].

Norm-Error and Absolute Error

In this research, model performance is evaluated using the error norms (L_2 and L_∞) and the absolute error. The L_2 norm is used to measure the mean squared error of the numerical solution relative to the exact solution over the entire computational domain. In contrast, the L_∞ norm measures the maximum error between the numerical and exact solutions over the entire computational domain. Mathematically, the L_2 and L_∞ norms can be expressed as follows:

$$L_2 = \sqrt{\sum_{i=1}^N |C_i^{exact} - C_i^{num}|^2} \quad (5)$$

$$L_\infty = \max_i |C_i^{exact} - C_i^{num}| \quad (6)$$

Where C_i^{num} is the concentration value of the numerical solution, and C_i^{exact} is the concentration value of the exact solution, and N is the number of computational points.

Furthermore, this study also uses the absolute error to measure the performance of the proposed model. The following equation is used to calculate the average absolute error.

$$E = |C_i^{exact} - C_i^{num}| \quad (7)$$

The error values of the proposed numerical model are then compared with the norm-error values of other numerical models from

previous studies on the same case. This comparison is expected to demonstrate the superiority or contribution of the proposed model in improving the quality of one-dimensional pollutant transport simulations.

RESULT AND DISCUSSION

First Case

The first case used to assess the model’s performance in simulating pollutant transport phenomena is the one-dimensional pure advection case. In this case, pollutant transport occurs solely due to advection or the flow along the channel. This case has been extensively studied by previous researchers, allowing comparison of the simulation results from the proposed numerical model with those of other numerical models.

In the first case, the computational domain is depicted as a 9000 m long channel (L) with a flat channel bed slope and a constant cross-section. The channel is assumed to flow at a fluid velocity of 0.5 m/s. The initial pollutant distribution is then modeled using equation (8) with a distribution parameter ρ of 264 m and a peak concentration (x_0) located at 2000 m. Furthermore, Dirichlet and Neumann boundary conditions are applied to this case. At the beginning of the channel ($x = 0$), the pollutant concentration is set to zero, while at the end of the channel ($x = 9000$ m), the concentration gradient is set to zero. These two boundary conditions can also be expressed mathematically in equations (9) and (10).

$$C(x, t) = 10 \exp\left(\frac{-(x - x_0 - U \cdot t)^2}{2\rho^2}\right) \quad (8)$$

$$-D \frac{\partial C}{\partial x}(L, t) = 0 \quad (9)$$

$$C(0, t) = 0 \quad (10)$$

Where ρ is a Gaussian distribution, and x_0 is the initial location of the pollutant peak. Based on previous research, the ρ and x_0 used are 264 m and 2000 m, respectively.

The simulation results show that the concentration profile of the proposed model

provides good agreement with the exact solution, as shown in Figure 2. In this case, the parameter value α is 0.99, obtained by trial and error. This value was chosen because it provides stable and accurate results. Based on Table 1, the peak concentration at 9600 s is 9.999 mg/L, which is very close to the exact solution of 10,000 mg/L. This difference indicates that the model has good accuracy in simulating this pure advection case. Based on the norm-error, the proposed model also provides L_2 and L_∞ values of 0.048 and 0.011, respectively, which are much smaller than those of other models from previous studies, such as FEMSLF, FEMQS, and the FTCS with Hansen Filter. Although the CD6 model still has a smaller norm-error, the proposed model still shows excellent performance with a relatively small error rate.

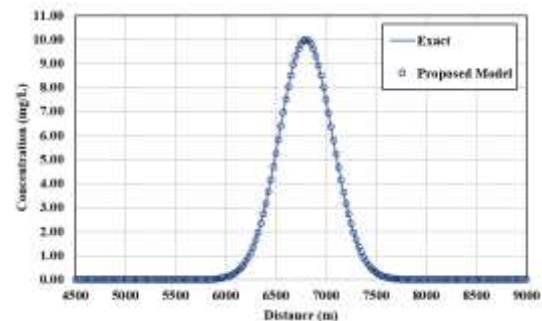


Figure 2. First Case Simulation Results For $t = 9600$ s ($\Delta x = 25$ m and $\Delta t = 50$ s)

Table 1. Norm-Error for 1D Advection Problem (CFL = 1)

Numerical Model	Peak Concentration Value (mg/L)	L_2	L_∞
Exact Solution	10.000		
FEMSLF [15]	9.943	7.881	0.377
FEMQS [15]	9.961	7.908	0.379
CD6 [16]	10.000	0.003	0.001
FTCS with Hansen Filter [22]	9.971	1.632	0.392
Proposed Model	9.999	0.048	0.011

Second Case

The second test case in this study is a one-dimensional advection-diffusion case. In this case, pollutant distribution involves two mechanisms (advection and diffusion)

simultaneously, thereby introducing greater complexity than in the first case. In this second simulation, several physical parameters are used to model pollutant transport. A 100 m-long channel (L) is used to represent the computational domain in which pollutant transport occurs. Within the channel, there is a flow with a velocity (U) of 0.01 m/s, which contributes to advection. The diffusion process in this simulation is modeled using a dispersion coefficient (D) of 0.0002 m²/s. Next, an exact solution from previous research was used to validate the proposed model. The analytical solution is expressed in equation (11). Using this equation, the initial conditions for this case can also be obtained by substituting $t = 0$ s into the equation.

$$C(x, t) = \frac{1}{2} \operatorname{erfc}\left(\frac{x - Ut}{\sqrt{4Dt}}\right) + \frac{1}{2} \exp\left(\frac{Ux}{D}\right) \operatorname{erfc}\left(\frac{x + Ut}{\sqrt{4Dt}}\right) \quad (11)$$

Meanwhile, the boundary conditions are Dirichlet at $x = 0$ and Neumann at $x = L$. These two boundary conditions are expressed in equations (12) and (13).

$$-D \frac{\partial C}{\partial x}(L, t) = 0 \quad (12)$$

$$C(0, t) = 1 \quad (13)$$

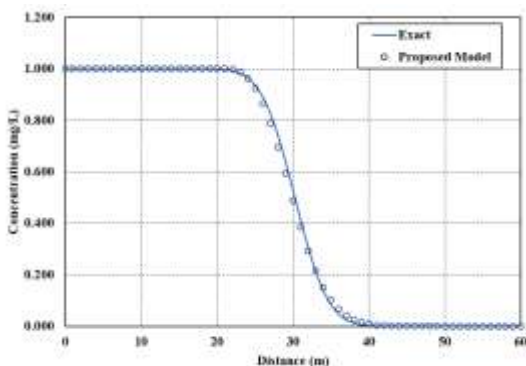


Figure 3. Second Case Simulation Results For $t = 3000$ s ($\Delta x = 1$ m and $\Delta t = 20$ s)

Figure 3 compares the exact solution and simulation results for the proposed model with $\alpha = 0.04$ and $\Delta t = 20$ s. The figure shows that the proposed model produces a concentration profile that closely matches the exact solution,

with no significant numerical oscillations. The figure shows that the model produces good and stable results. Furthermore, the norm-error values presented in Table 2 also show good results. The table shows that the FTCS has L_2 and L_∞ norm errors of 0.1412 and 0.0587, respectively. Furthermore, the FTCS model with the Hansen filter yields a lower error rate, with $L_2 = 0.0964$ and $L_\infty = 0.0448$, indicating that the Hansen filter can improve the accuracy of the FTCS model. Meanwhile, the modified Lax model with the Hansen filter shows performance that depends on the time step. For the simulation with $\Delta t = 10$ s, the model has L_2 and L_∞ norm-error rates of 0.2677 and 0.0892, respectively, which are larger than those of the FTCS model, both with and without the Hansen filter. However, when $\Delta t = 20$ s, the error rates of L_2 and L_∞ become 0.0948 and 0.0422, respectively. These values are better than conventional FTCS and FTCS with Hansen Filter reported in previous studies [22].

Table 2. Comparison of Simulation Results for the Second Case

x (m)	Exact	FTCS	FTCS with Hansen Filter	Proposed Model ($\alpha = 0.04$)	
				dt = 10 s	dt = 20 s
0	1.000	1.000	1.000	1.000	1.000
21	0.996	1.008	1.006	0.978	1.002
22	0.991	1.017	1.007	0.962	0.998
23	0.982	1.021	1.000	0.939	0.987
24	0.964	1.009	0.980	0.906	0.963
25	0.934	0.975	0.941	0.864	0.924
26	0.889	0.911	0.880	0.810	0.866
27	0.823	0.821	0.798	0.747	0.789
28	0.738	0.709	0.700	0.675	0.696
29	0.636	0.587	0.591	0.597	0.594
30	0.523	0.464	0.481	0.516	0.488
31	0.408	0.352	0.377	0.435	0.386
32	0.301	0.255	0.284	0.358	0.294
33	0.208	0.177	0.206	0.287	0.215
34	0.135	0.118	0.144	0.224	0.152
35	0.082	0.075	0.097	0.171	0.103
36	0.046	0.046	0.063	0.127	0.067
37	0.024	0.027	0.039	0.091	0.042
L_2		0.1412	0.0964	0.2677	0.0948

L_∞	0.0587	0.0448	0.0892	0.0422
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Third Case

The final case simulated in this study is the one-dimensional Gaussian pulse case. This case follows a Gaussian distribution, enabling clear analysis of pollutant dispersion over time. This case is widely used as a standard test case in the development of numerical models for the advection-diffusion equation because it has an explicitly known analytical solution. In this simulation, the parameters used include a flow velocity of 0.8 m/s, a dispersion coefficient of 0.005 m²/s, and a channel length of 9 m. The exact solution and initial conditions in this case can be obtained by equation (14). Meanwhile, the boundary conditions for both ends of the channel are expressed by equations (15) and (16).

$$C(x, t) = \frac{1}{\sqrt{4t + 1}} \exp\left(-\frac{(x - 1 - Ut)^2}{D(4t + 1)}\right) \quad (14)$$

$$C(0, t) = \frac{1}{\sqrt{4t + 1}} \exp\left(-\frac{(1 - Ut)^2}{D(4t + 1)}\right) \quad (15)$$

$$C(9, t) = \frac{1}{\sqrt{4t + 1}} \exp\left(-\frac{(8 - Ut)^2}{D(4t + 1)}\right) \quad (16)$$

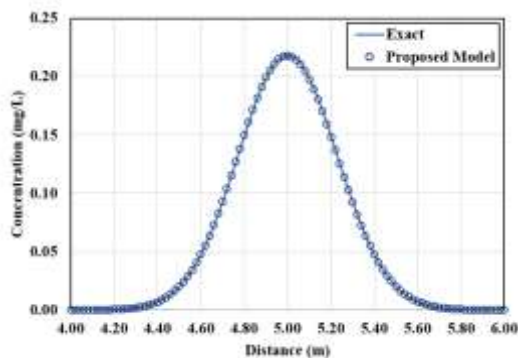


Figure 4. Third Case Simulation Results For $t = 5$ s

In this case, the performance of the proposed model was evaluated by comparing the simulation results with the exact solution (Figure 4). The numerical parameters used to compare the simulation results were $\Delta x = 0.02$ m, $\Delta t = 0.01$ s, and $\alpha = 0.15$. These values were obtained through trial and error. Parameter selection also considers the Courant–Friedrichs–Lewy (CFL) number to be no greater than 1. In this case, smaller Δx and Δt

will yield a more stable and accurate numerical solution, though they increase computational cost. The comparison results showed that the pollutant concentration profile from the proposed model agreed very well with the analytical solution. This agreement indicates that the numerical model accurately represents the pollutant transport process.

Table 3. Absolute Error Value

Distance (m)	Concentration (mg/L)			Absoluter Error	
	Exact	Proposed Model	FTCS with Hansen Filter [22]	Proposed Model	FTCS with Hansen Filter [22]
3.5	0.00E+00	0.00E+00	0.00E+00	6.66E-11	6.66E-11
4.0	1.60E-05	1.30E-05	1.20E-05	3.25E-6	3.61E-6
4.5	2.02E-02	2.00E-02	1.99E-02	2.02E-4	2.84E-4
5.0	2.18E-01	2.18E-01	2.18E-01	8.76E-5	3.70E-6
5.5	2.02E-02	2.04E-02	2.04E-02	2.11E-4	2.71E-4
6.0	1.60E-05	1.90E-05	2.00E-05	2.59E-6	4.39E-6
6.5	0.00E+00	0.00E+00	0.00E+00	6.59E-11	1.55E-10

In addition to visual analysis, model performance was evaluated based on absolute errors between the exact solution and the numerical simulation results at several computational points, as presented in Table 3. The table also presents a comparison of the absolute error values of the proposed model and the FTCS model with a Hansen Filter. Based on the table, the proposed numerical model has a relatively low absolute error. At the peak concentration point ($x = 5$ m), the numerical model’s concentration is 0.218130 mg/L, and the exact concentration is 0.218218 mg/L. These two values have a very small difference expressed in an absolute error of 8.76×10^{-5} . This error value indicates that the numerical model accurately captures the peak pollutant concentration without significant numerical oscillations. Table 3 also shows that, overall, the proposed numerical model provides a smaller absolute error than the FTCS model with the Hansen Filter. Overall, the simulation and comparison results indicate that the proposed model accurately represents the distribution of pollutant concentrations while maintaining numerical stability.

CONCLUSION

This study develops a one-dimensional numerical model for pollutant transport using the modified Lax Scheme combined with the Hansen numerical filter. The performance of the proposed model is further evaluated across three cases of pollutant transport: pure advection, advection-diffusion, and Gaussian Pulse. The simulation results for the three cases indicate that the proposed model provides good agreement with the exact solution, without significant numerical oscillations. The comparison between the exact solution and the proposed model shows that the model errors are relatively small, as indicated by the low L_2 and L_∞ norm errors and the absolute error. Furthermore, the proposed numerical model generally offers greater accuracy than other models, such as FTCS, FTCS with Hansen Filter, FEMLSF, and FEMQSF. The simulation results also highlight the importance of selecting the appropriate value of α -parameter in the proposed model to achieve accurate and stable results. An appropriate choice of α helps reduce numerical oscillations while maintaining the accuracy of the pollutant concentration profile. In addition to accuracy, the proposed model has the advantage of a simple numerical formulation, making it easy to implement and computationally efficient compared to models with more complex formulations. This capability makes the proposed model an effective alternative for one-dimensional simulations of pollutant transport that involve advection and diffusion. However, this study is limited to one-dimensional cases and controlled test conditions. Therefore, further research is recommended to extend the proposed model to two- and three-dimensional pollutant transport problems and more complex flow conditions, and then validate it using experimental or field data.

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